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# Quantum dynamics of a generalized Coleman-Hepp model

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**Abstract.** Studies on a generalized Coleman-Hepp model are done on the basis of a spin coherent state representation and a transformation property of the model Hamiltonian. Namely, transforming the original model Hamiltonian into a simpler form, we can determine time evolution of the whole system by successive applications of rotation operators in a spinor space. Dynamics of detector spins as well as that of an incident particle are fully discussed. Explicit numerical evaluations are also performed. Relevance of our solution to a generalized Cini model is also briefly mentioned.

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## 1 Introduction

Decoherence in quantum systems has played important roles from the very beginning of the quantum mechanics mainly in connection with measurement theory [1,2].

Meanwhile, the decoherence phenomena have long been studied in the field of nonequilibrium statistical physics.

For instance, since the discovery of nuclear magnetic resonance [3,4], the decoherence is classified into two categories, namely, one is the so-called longitudinal relaxation in which nuclear spins lose their energies in interaction with reservoir, and the other is the transverse relaxation where phase coherence of the spins is lost due to the interaction with the reservoir. Sometimes the former is called $T_1$ process whereas the latter $T_2$ (dephasing) process. These relaxation processes are incorporated into a simple phenomenological Bloch equation [5].

Subsequently, microscopic basis and generalization of the Bloch equation are given [6–10]. The theoretical framework on spin relaxation [11–13] has wide applicability and has been used in quantum electronics [14,15], quantum optics [16–18], photophysics of solids and so on. These theories are formulated within lower order perturbational method and their validity is confined to the narrowing limit where correlation time of the reservoir is very short compared with the characteristic time of relaxation.

When the relevant system is strongly coupled with its environment, the conventional perturbation approach becomes no longer valid, and therefore we must take into account the interaction as a whole. However, this is a formidable task to perform in practical applications. To our knowledge, only a few stochastic models are exactly solvable: Kubo-Anderson model of a random frequency modulation [19,20], a dielectric relaxation model with inertia effect [21], and a low (zero) field resonance model of spin relaxation [22,23] which is a special case of the original low field model [24].

In contrast to the above mentioned traditional relaxation (decoherence) theories, there exists a special but nonetheless important solvable model of the quantum decoherence. This is due to Coleman and Hepp [25] who introduced the model in order to examine measurement process of quantum mechanics. The original model consists of an incident particle and an array of spins which are regarded as a detector. When the particle approaches a detector spin, the former exerts force on the latter destroying initial memory and *vice versa*. There have been several attempts to examine the original model itself [26,27], and variant [28] or generalization [29–32] of the model. In these articles, main interest lies in the quantum mechanical measurement process and/or relaxation (decoherence) process. Especially, emergence of a Wiener process [31] reflects essential relevance of the model to the irreversible phenomena mentioned earlier. Furthermore, spin relaxation (dephasing) process of the incident particle was also analyzed [32].

Then, our purpose in this paper is twofold: one is to give a simple theoretical framework to analyze a generalized version of the Coleman-Hepp (C-H) model. This can be done on the basis of a transformation property in reference [32] and a spin coherent state representation [33,34] with which spin algebra of the detector spins is performed

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only in $2 \times 2$ space even for arbitrary spin magnitude of $S$. The other purpose is to determine explicitly dynamical process of the detector spins as well as that of an incident particle. This is because of little detailed studies on the dynamical process of the detector. To our knowledge, only a single reference [29] exists on the stochastic process of the detector. In addition, it is important to find a rigorous solution for a quantum mechanical decoherence (relaxation) model other than the stochastic models mentioned above. Namely, we determine time evolution of the whole system and solve for averaged quantities of the relevant particle and the detector. Numerical evaluation of these quantities is also performed.

## 2 Preliminaries

### 2.1 Model Hamiltonians and transformation properties

Now let us introduce an extended version of the Coleman-Hepp model which is composed of an incident particle and a detector. The particle is characterized by a position operator $X$, a momentum operator $P$ and a spin $\mathbf{I}$ of magnitude 1/2. A set of $N$ spins $\{\mathbf{S}_l\}$, $(l = 1, 2, ..., N)$ constitutes the detector; the $l$th spin is located at the position $x_l$.

Thus, our system can be described by the Hamiltonian of the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{P}_+\mathcal{H}_1 \tag{1}$$

where

$$\mathcal{H}_0 = \mathcal{H}_\mathrm{I} + \mathcal{H}_\mathrm{D} \tag{2}$$

with

$$\mathcal{H}_\mathrm{I} = vP + \hbar\omega_\mathrm{I} I^z \tag{3}$$

and

$$\mathcal{H}_\mathrm{D} = \sum_{l=1}^{N} \hbar\omega_l S_l^z. \tag{4}$$

The Hamiltonian of the incident particle is given by $\mathcal{H}_\mathrm{I}$ whereas that of the detector by $\mathcal{H}_\mathrm{D}$; $v$ and $\hbar\omega_\mathrm{I}$ are the velocity and the spin energy of the particle, respectively, and $\hbar\omega_l$ the energy of the $l$th detector spin. These observables satisfy $[X, P] = \mathrm{i}\hbar$, $[S_l^x, S_{l'}^y] = \mathrm{i}\delta_{l,l'} S_l^z$ and cyclic permutations thereof.

An interaction Hamiltonian between the incident particle and the detector is given by

$$\mathcal{H}_1 = \sum_{l=1}^{N} \hbar\boldsymbol{\Omega}_l(X, x_l) \cdot \mathbf{S}_l \tag{5}$$

where

$$\boldsymbol{\Omega}_l(X, x_l) = \Omega_l(X - x_l)\left(\cos\left(\omega_l X/v\right), \sin\left(\omega_l X/v\right), 0\right) \tag{6}$$

is due to an effective field on the $l$th detector spin when the particle enters into a neighborhood region around the position $x_l$.

Then the interaction Hamiltonian takes the form

$$\mathcal{H}_1 = \frac{1}{2}\sum_l \hbar\Omega_l(X - x_l)\left\{\mathrm{e}^{\mathrm{i}\omega_l X/v} S_l^- + \mathrm{e}^{-\mathrm{i}\omega_l X/v} S_l^+\right\} \tag{7}$$

where we have put

$$S_l^\pm = S_l^x \pm \mathrm{i}S_l^y. \tag{8}$$

In (1), the projection operator $\mathcal{P}_+$ is defined by

$$\mathcal{P}_+ = \frac{1}{2} + I^z \tag{9}$$

which takes value of 1(0) when the particle spin is up(down).

For later convenience we also define

$$\mathcal{P}_- = \frac{1}{2} - I^z \tag{10}$$

with an idempotent relation,

$$\mathcal{P}_\pm^2 = \mathcal{P}_\pm \tag{11}$$

and

$$\mathcal{P}_+ + \mathcal{P}_- = 1 \tag{12}$$

ensuring $\mathcal{P}_\pm$ to be the projection operators.

Next, we briefly summarize transformation properties of the Hamiltonians and the time evolution operator $\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}$ introduced in reference [32].

The second term on the right hand side of (1) gives a restriction that the interaction is effective only when the incident particle is in the up-spin state.

Therefore, we have the following simple relation:

$$\mathcal{H}\mathcal{P}_+ = (\mathcal{H}_0 + \mathcal{H}_1)\mathcal{P}_+ \tag{13}$$

and

$$\mathcal{H}\mathcal{P}_- = \mathcal{H}_0\mathcal{P}_- \tag{14}$$

which yield

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}\mathcal{P}_+ = \mathrm{e}^{-\mathrm{i}(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}\mathcal{P}_+ \tag{15}$$

and

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}\mathcal{P}_- = \mathrm{e}^{-\mathrm{i}\mathcal{H}_0 t/\hbar}\mathcal{P}_-. \tag{16}$$

From equations (15, 16), we have

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar} = \mathrm{e}^{-\mathrm{i}(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}\mathcal{P}_+ + \mathrm{e}^{-\mathrm{i}\mathcal{H}_0 t/\hbar}\mathcal{P}_- \tag{17}$$

where use has been made of (12).

Next we examine transformation properties of the Hamiltonian under rotations.

Let a rotational operator in the spin space of the detector be $\mathcal{D}(\{\phi_l\})$. Especially, rotations around $z$-axis are given by

$$\mathcal{D}^z(\{\phi_l\}) = \prod_l \mathcal{D}_l^z(\phi_l) \tag{18}$$

where

$$\mathcal{D}_l^z(\phi_l) = \mathrm{e}^{-\mathrm{i}\phi_l S_l^z} \tag{19}$$

which transforms $S_l^{\pm}$ and $vP$ as

$$\mathcal{D}_l^z(\omega_l X/v) S_l^{\pm} \mathcal{D}_l^z(\omega_l X/v)^\dagger = S_l^{\pm} \mathrm{e}^{\mp \mathrm{i}\omega_l X/v} \tag{20}$$

and

$$\mathcal{D}_l^z(\omega_l X/v) vP \mathcal{D}_l^z(\omega_l X/v)^\dagger = vP + \hbar\omega_l S_l^z. \tag{21}$$

Using above relations, we find immediately

$$\mathcal{D}^z(\{\omega_l X/v\}) \mathcal{H}_0{}' \mathcal{D}^z(\{\omega_l X/v\})^\dagger = \mathcal{H}_0 \tag{22}$$

and

$$\mathcal{D}^z(\{\omega_l X/v\}) \mathcal{H}_1{}' \mathcal{D}^z(\{\omega_l X/v\})^\dagger = \mathcal{H}_1 \tag{23}$$

where we have defined

$$\mathcal{H}_0{}' = vP + \hbar\omega_{\mathrm{I}} I^z \tag{24}$$

and

$$\mathcal{H}_1{}' = \sum_l \hbar\Omega_l(X - x_l) S_l^x. \tag{25}$$

Thus the time evolution operator (17) is rewritten in terms of the simpler Hamiltonians $\mathcal{H}_0{}'$ and $\mathcal{H}_1{}'$:

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar} = \mathcal{D}^z(\{\omega_l X/v\}) \left\{ \mathrm{e}^{-\mathrm{i}(\mathcal{H}_0{}' + \mathcal{H}_1{}')t/\hbar} \mathcal{P}_+ + \mathrm{e}^{-\mathrm{i}\mathcal{H}_0{}' t/\hbar} \mathcal{P}_- \right\} \mathcal{D}^z(\{\omega_l X/v\})^\dagger. \tag{26}$$

It is more convenient to extract the interaction part in the evolution operator. Namely, we write

$$\mathrm{e}^{-\mathrm{i}(\mathcal{H}_0{}' + \mathcal{H}_1{}')t/\hbar} = \mathrm{e}^{-\mathrm{i}\mathcal{H}_0{}' t/\hbar} V(t) \tag{27}$$

obtaining an equation for $V(t)$:

$$\dot{V}(t) = -\frac{\mathrm{i}}{\hbar} \hat{\mathcal{H}_1}{}'(t) V(t) \tag{28}$$

where

$$\hat{\mathcal{H}_1}{}'(t) = \mathrm{e}^{\mathrm{i}\mathcal{H}_0{}' t/\hbar} \mathcal{H}_1{}' \mathrm{e}^{-\mathrm{i}\mathcal{H}_0{}' t/\hbar} \tag{29}$$

$$= \sum_l \hbar\Omega_l(X + vt - x_l) S_l^x. \tag{30}$$

Because of the property that

$$\left[\hat{\mathcal{H}_1}{}'(t), \hat{\mathcal{H}_1}{}'(t')\right] = 0 \tag{31}$$

for arbitrary $t$ and $t'$, we can solve (28) easily to get

$$V(t) = \exp\left[-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t'\, \hat{\mathcal{H}_1}{}'(t')\right] V(0) \tag{32}$$

$$= \exp\left[-\mathrm{i}\sum_l \Theta_l(X;t) S_l^x\right] \tag{33}$$

where $V(0) = 1$ and we have put

$$\Theta_l(x;t) = \int_0^t \mathrm{d}t'\, \Omega_l(x + vt' - x_l). \tag{34}$$

Thus, from (26), the time evolution of the whole system is determined by

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar} = \mathcal{D}^z(\{\omega_l X/v\}) \{ \mathrm{e}^{-\mathrm{i}\mathcal{H}_0{}' t/\hbar} V(t) \mathcal{P}_+ + \mathrm{e}^{-\mathrm{i}\mathcal{H}_0{}' t/\hbar} \mathcal{P}_- \} \mathcal{D}^z(\{\omega_l X/v\})^\dagger \tag{35}$$

which will be used in our subsequent calculations.

## 2.2 Spin coherent state representation

In order to describe details of spin systems, the spin coherent state affords a powerful tool. That is, we would find a straightforward way of manipulations using the state. Without it, rather cumbersome calculations are inevitable. Thus, we briefly summarize the known results of the spin coherent state [33,34].

### 2.2.1 Spin coherent state

Let us first represent the spin operators in terms of the Schwinger bosons [35,36]:

$$S^{\pm} \equiv b_{\pm}^\dagger b_{\mp}, \tag{36}$$

$$S^z \equiv \frac{1}{2}(N_+ - N_-), \tag{37}$$

and

$$N_{\pm} \equiv b_{\pm}^\dagger b_{\pm} \tag{38}$$

with

$$[b_{\pm}, b_{\pm}^\dagger] = 1. \tag{39}$$

A simultaneous eigenstate of $N_+$ and $N_-$ are of the form

$$N_{\pm}|n_+, n_-\rangle = n_{\pm}|n_+, n_-\rangle \tag{40}$$

where

$$|n_+, n_-\rangle \equiv |n_+\rangle \otimes |n_-\rangle \tag{41}$$

whose constituent states $|n_{\pm}\rangle$ satisfy

$$b_{\pm}^\dagger|n_{\pm}\rangle = \sqrt{n_{\pm} + 1}|n_{\pm} + 1\rangle \tag{42}$$

and

$$b_\pm|n_\pm\rangle = \sqrt{n_\pm}|n_\pm - 1\rangle \tag{43}$$

with

$$|n_+, n_-\rangle = \frac{\left(b_+^\dagger\right)^{n_+}\left(b_-^\dagger\right)^{n_-}}{\sqrt{n_+!}\sqrt{n_-!}}|0,0\rangle. \tag{44}$$

Finally, we have

$$S^\pm|S,m) = \sqrt{(S\mp m)(S\pm m+1)}|S, m\pm 1), \tag{45}$$

$$S^z|S,m) = m|S,m) \tag{46}$$

where we write

$$|S,m) \equiv |n_+ = S+m\rangle \otimes |n_- = S-m\rangle. \tag{47}$$

Thus, the product number state is seen to be nothing but the angular momentum state.

Then, it is natural to introduce a coherent state for spin (angular momentum) by extending the usual boson coherent state:

$$b_\pm|\mathbf{z}\rangle = z_\pm|\mathbf{z}\rangle \tag{48}$$

with

$$|\mathbf{z}\rangle = |z_+\rangle \otimes |z_-\rangle \tag{49}$$

where $|z_+\rangle$ ($|z_-\rangle$) is the boson coherent state for the annihilation operator $b_+$ ($b_-$).

We call $|\mathbf{z}\rangle$ the "spin coherent state" which has several expressions:

$$|\mathbf{z}\rangle = D(\mathbf{z})|\mathbf{0}\rangle \tag{50}$$

$$= e^{-|\mathbf{z}|^2/2}\sum_{n_+=0}^{\infty}\sum_{n_-=0}^{\infty}\frac{z_+^{n_+}z_-^{n_-}}{\sqrt{n_+!}\sqrt{n_-!}}|n_+, n_-\rangle \tag{51}$$

$$= e^{-|\mathbf{z}|^2/2}\sum_{S=0}^{\infty}\sum_{m=-S}^{S}\frac{z_+^{S+m}z_-^{S-m}}{\sqrt{(S+m)!}\sqrt{(S-m)!}}|S,m) \tag{52}$$

where

$$D(\mathbf{z}) = \exp\left[\mathbf{z}\mathbf{b}^\dagger - \mathbf{z}^*\mathbf{b}\right]. \tag{53}$$

In these expressions, the bold-faced quantities should be flexibly recognized as two-component column or row vectors. Hence we have, for instance,

$$\mathbf{z}\mathbf{b}^\dagger = (z_+, z_-)\begin{pmatrix} b_+^\dagger \\ b_-^\dagger \end{pmatrix} \tag{54}$$

$$= z_+b_+^\dagger + z_-b_-^\dagger. \tag{55}$$

We have also written the "vacuum" as

$$|\mathbf{0}\rangle \equiv |n_+ = 0\rangle \otimes |n_- = 0\rangle. \tag{56}$$

Inner product of the spin coherent states is given by

$$\langle\mathbf{z}|\mathbf{z}'\rangle = \exp\left[\mathbf{z}^*\mathbf{z}' - \frac{1}{2}(|\mathbf{z}|^2 + |\mathbf{z}'|^2)\right]. \tag{57}$$

#### 2.2.2 Transformation properties of $|\mathrm{z}\rangle$ under rotations

We already introduced the rotation operator $\mathcal{D}^z(\phi) = e^{-i\phi S^z}$ in (19). Transformation properties of the spin coherent state is very simple under rotations. That is, the states $\{|\mathbf{z}\rangle\}$ transform among themselves.

Explicitly, let us consider effect of $\mathcal{D}^z(\phi)$ on $|\mathbf{z}\rangle$:

$$\mathcal{D}^z(\phi)|\mathbf{z}\rangle = e^{-(|z_+|^2+|z_-|^2)/2} \times\left(\sum_{n_+}\frac{\left(e^{-i\phi/2}z_+\right)^{n_+}}{\sqrt{n_+!}}|n_+\rangle\right) \otimes\left(\sum_{n_-}\frac{\left(e^{i\phi/2}z_-\right)^{n_-}}{\sqrt{n_-!}}|n_-\rangle\right) \tag{58}$$

$$= |\mathbf{z}'\rangle \tag{59}$$

where

$$\mathbf{z}' = \begin{pmatrix} z_+{}' \\ z_-{}' \end{pmatrix} \tag{60}$$

$$= \begin{pmatrix} e^{-i\phi/2}z_+ \\ e^{i\phi/2}z_- \end{pmatrix}. \tag{61}$$

We can rewrite above expression as follows:

$$\mathbf{z}' = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}\begin{pmatrix} z_+ \\ z_- \end{pmatrix} \tag{62}$$

$$= \mathcal{R}^z(\phi)\mathbf{z} \tag{63}$$

where

$$\mathcal{R}^z(\phi) = e^{-i\phi\sigma^z/2} \tag{64}$$

with the Pauli matrix

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{65}$$

We have thus

$$\mathcal{D}^z(\phi)|\mathbf{z}\rangle = |\mathcal{R}^z(\phi)\mathbf{z}\rangle. \tag{66}$$

Similarly, the following relations are obtained:

$$\mathcal{D}^\mu(\phi)|\mathbf{z}\rangle = |\mathcal{R}^\mu(\phi)\mathbf{z}\rangle \qquad (\mu = x,y,z) \tag{67}$$

where

$$\mathcal{D}^\mu(\phi) = e^{-i\phi S^\mu} \tag{68}$$

and

$$\mathcal{R}^\mu(\phi) = e^{-i\phi\sigma^\mu/2}. \tag{69}$$

More generally, a rotation of angle $\phi$ of the spin coherent state $|\mathbf{z}\rangle$ around an axis specified by a unit vector $\hat{\mathbf{n}}$

is equivalent to the same rotation of the two-component vector $\mathbf{z}$:

$$\mathcal{D}(\phi, \hat{\mathbf{n}})|\mathbf{z}\rangle = |\mathcal{R}(\phi, \hat{\mathbf{n}})\mathbf{z}\rangle \tag{70}$$

where

$$\mathcal{D}(\phi, \hat{\mathbf{n}}) = \mathrm{e}^{-\mathrm{i}\phi\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}/2}. \tag{71}$$

These will be used in the following development.

### 2.2.3 Basic spin coherent state and Bloch state

An important spin coherent state is generated from a special spin coherent state $|z_+ = z, z_- = 0\rangle$ by an Euler rotation, $z$ being a complex number:

$$|\mathbf{z}\rangle = \mathrm{e}^{-\mathrm{i}\phi S^z}\mathrm{e}^{-\mathrm{i}\theta S^y}|z, 0\rangle \tag{72}$$

where $\mathbf{z}$ is given by

$$\begin{pmatrix} z_+ \\ z_- \end{pmatrix} = \begin{pmatrix} z\mathrm{e}^{-\mathrm{i}\phi/2}\cos\theta/2 \\ z\mathrm{e}^{\mathrm{i}\phi/2}\sin\theta/2 \end{pmatrix}. \tag{73}$$

In obtaining above expressions, we have used (67). Next, we rewrite (73) in an alternative form.

From (52) we have

$$|z, 0\rangle = \mathrm{e}^{-|z|^2/2}\sum_{S=0}^{\infty}\frac{z^{2S}}{\sqrt{(2S)!}}|S, S) \tag{74}$$

which clearly shows that the special spin coherent state $|z, 0\rangle$ is a superposition of the "spin up state" $|S, S)$. This is the reason why we set $z_- = 0$ in specifying the state $|z, 0\rangle$. From equations (72, 74) we find

$$|\mathbf{z}\rangle = \mathrm{e}^{-|z|^2/2}\sum_{S=0}^{\infty}\frac{z^{2S}}{\sqrt{(2S)!}}|S;\theta,\phi\rangle \tag{75}$$

where

$$|S;\theta,\phi\rangle = \mathrm{e}^{-\mathrm{i}\phi S^z}\mathrm{e}^{-\mathrm{i}\theta S^y}|S, S) \tag{76}$$

is called the Bloch state [37,38].

Roughly speaking, in the Bloch state, a spin points to the direction specified by the polar angle $\theta$ and the azimuthal angle $\phi$.

Let $A(\mathbf{S})$ be an operator function of $\mathbf{S}$ whose magnitude is $S$. Then we have

$$\langle\mathbf{z}|A(\mathbf{S})|\mathbf{z}\rangle = \mathrm{e}^{-|z|^2}\sum_{S=0}^{\infty}\frac{|z|^{4S}}{(2S)!}\langle S;\theta,\phi|A(\mathbf{S})|S;\theta,\phi\rangle. \tag{77}$$

Information on a fixed subspace of spin magnitude $S$ is contained in the matrix element in the right hand side of (77). Superiority of the use of $|\mathbf{z}\rangle$ over direct use of $|S;\theta,\phi\rangle$ is clearly seen in our subsequent developments.

## 3 Quantum dynamical processes

In the present section let us proceed to obtain explicit basic expressions for several observables.

We confine our discussions to a simple initial condition on the wave function of the whole system.

Let the initial state of the whole system be $|\Psi(0)\rangle\rangle$:

$$|\Psi(0)\rangle\rangle = |I\rangle \otimes |\psi\rangle \otimes |\{\mathbf{z}_l\}\rangle \tag{78}$$

where

$$|I\rangle = a|+\rangle + b|-\rangle, \tag{79}$$

$$|\psi\rangle = \int \mathrm{d}x\,\psi(x)|x\rangle, \tag{80}$$

and

$$|\{\mathbf{z}_l\}\rangle = \prod_{l=1}^{N}|\mathbf{z}_l\rangle \tag{81}$$

are, respectively, the spin state and orbital state of the incident particle, and the detector state composed of $N$ spins.

The incident particle is assumed to have spin 1/2:

$$I^z|\pm\rangle = \pm\frac{1}{2}|\pm\rangle. \tag{82}$$

While, the orbital state is expressed by the eigenstate of $X$:

$$X|x\rangle = x|x\rangle. \tag{83}$$

The initial state of the detector is the product of the constituent spin coherent state of $N$ spins.

### 3.1 Dynamical process of an incident particle spin

We study transverse spin dynamical process of the incident particle determined by

$$\begin{aligned}\langle\langle I^-(t)\rangle\rangle &\equiv \langle\langle\Psi(0)|I^-(t)|\Psi(0)\rangle\rangle \\ &= \langle\langle\Psi(0)|\mathrm{e}^{\mathrm{i}\mathcal{H}t/\hbar}I^-\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}|\Psi(0)\rangle\rangle\end{aligned} \tag{84}$$

where

$$I^- = I^x - \mathrm{i}I^y. \tag{85}$$

Using (17) and a relation $I^-\mathcal{P}_- = 0$, we have

$$I^-\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar} = I^-\mathrm{e}^{-\mathrm{i}(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}\mathcal{P}_+. \tag{86}$$

With the conjugate relation of (17) and a relation $\mathcal{P}_+I^-\mathcal{P}_+ = 0$, we find

$$\mathrm{e}^{\mathrm{i}\mathcal{H}t/\hbar}I^-\mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar} = I^-\mathrm{e}^{\mathrm{i}\mathcal{H}_0t/\hbar}\mathrm{e}^{-\mathrm{i}(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}\mathcal{P}_+\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{I}}t} \tag{87}$$

which gives

$$\langle\langle I^-(t)\rangle\rangle = ab^*\langle +|\otimes\langle\psi|\otimes\langle\{\mathbf{z}_l\}|e^{i\mathcal{H}_0 t/\hbar} \times e^{-i(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}|\{\mathbf{z}_l\}\rangle\otimes|\psi\rangle\otimes|+\rangle e^{-i\omega_I t}. \quad (88)$$

Using equations (22, 23, 27) we obtain

$$e^{i\mathcal{H}_0 t/\hbar}e^{-i(\mathcal{H}_0+\mathcal{H}_1)t/\hbar} = \mathcal{D}^z(\{\omega_l X/v\})V(t)\mathcal{D}^z(\{\omega_l X/v\})^\dagger \quad (89)$$

where $V(t)$ is given by (33).

We have thus with the use of (80)

$$\langle\langle I^-(t)\rangle\rangle = ab^* e^{-i\omega_I t}\int dx\,|\psi(x)|^2 \times\langle\{\mathbf{z}_l\}|\prod_l \mathcal{D}_l^z(\omega_l x/v)V_l(x,t)\mathcal{D}_l^z(\omega_l x/v)^\dagger|\{\mathbf{z}_l\}\rangle \quad (90)$$

where

$$V_l(x,t) = \exp\left[-i\Theta_l(x;t)S_l^x\right] \quad (91)$$

$$= \mathcal{D}_l^x\left(\Theta_l(x;t)\right). \quad (92)$$

We note that the operator $V_l(x,t)$ is nothing but the rotation operator $\mathcal{D}_l^x(\phi)$ of (68).

Therefore, the matrix element on the right hand side of (90) can be evaluated easily with the help of (67):

$$\langle\mathbf{z}_l|\mathcal{D}_l^z(\omega_l x/v)\mathcal{D}_l^x\left(\Theta_l(x;t)\right)\mathcal{D}_l^z(\omega_l x/v)^\dagger|\mathbf{z}_l\rangle = \langle\mathbf{z}_l|\mathbf{z}_l'\rangle \quad (93)$$

where

$$\mathbf{z}'_l = \mathcal{R}_l^z(\omega_l x/v)\mathcal{R}_l^x\left(\Theta_l(x;t)\right)\mathcal{R}_l^z(-\omega_l x/v)\mathbf{z}_l = \begin{pmatrix} z_{l,+}\cos\dfrac{\Theta_l(x;t)}{2} - iz_{l,-}e^{-i\omega_l x/v}\sin\dfrac{\Theta_l(x;t)}{2} \\ -iz_{l,+}e^{i\omega_l x/v}\sin\dfrac{\Theta_l(x;t)}{2} + z_{l,-}\cos\dfrac{\Theta_l(x;t)}{2}\end{pmatrix}. \quad (94)$$

In obtaining (94) we used

$$\mathcal{R}_l^z(\phi) = e^{-i\phi\sigma_l^z/2} = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2}\end{pmatrix} \quad (95)$$

and

$$\mathcal{R}_l^x(\theta) = e^{-i\theta\sigma_l^x/2} = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2)\end{pmatrix}. \quad (96)$$

The inner product (93) is already given by (57) and hence we have

$$\langle\langle I^-(t)\rangle\rangle = ab^* e^{-i\omega_I t}\int dx\,|\psi(x)|^2\prod_l\langle\mathbf{z}_l|\mathbf{z}_l'\rangle \quad (97)$$

where

$$\langle\mathbf{z}_l|\mathbf{z}_l'\rangle = e^{-|z_l|^2}\exp\left[|z_l|^2\left\{\cos\frac{\Theta_l(x;t)}{2} - i\sin\theta_l\cos(\phi_l-\omega_l x/v)\sin\frac{\Theta_l(x;t)}{2}\right\}\right] \quad (98)$$

with

$$|z_l|^2 = |z_{l,+}|^2 + |z_{l,-}|^2. \quad (99)$$

In order to extract information on a fixed spin magnitude space, we further expand the right hand side of (98):

$$\langle\mathbf{z}_l|\mathbf{z}_l'\rangle = \langle\mathbf{z}_l|\mathcal{D}_l^z(\omega_l x/v)\mathcal{D}_l^x(\Theta_l(x;t))\mathcal{D}_l^z(-\omega_l x/v)|\mathbf{z}_l\rangle \quad (100)$$

$$= e^{-|z_l|^2}\sum_{S_l=0,\frac{1}{2},\cdots}^{\infty}\frac{|z_l|^{4S_l}}{(2S_l)!}\left[\cos\frac{\Theta_l(x;t)}{2} - i\sin\theta_l\cos(\phi_l-\omega_l x/v)\sin\frac{\Theta_l(x;t)}{2}\right]^{2S_l} \quad (101)$$

which is a special case of the formula (77) for the operator

$$A(\mathbf{S}_l) = \mathcal{D}_l^z(\omega_l x/v)\mathcal{D}_l^x\left(\Theta_l(x;t)\right)\mathcal{D}_l^z(-\omega_l x/v) \quad (102)$$

yielding an expression

$$\langle S_l;\theta_l,\phi_l|\mathcal{D}_l^z(\omega_l x/v)\mathcal{D}_l^z\left(\Theta_l(x;t)\right)\mathcal{D}_l^z(-\omega_l x/v)|S_l;\theta_l,\phi_l\rangle = \left[\cos\frac{\Theta_l(x;t)}{2} - i\sin\theta_l\cos(\phi_l-\omega_l x/v)\sin\frac{\Theta_l(x;t)}{2}\right]^{2S_l}. \quad (103)$$

Consequently, for the initial condition,

$$|\Psi(0)\rangle = |I\rangle\otimes|\psi\rangle\otimes|\{S_l;\theta_l,\phi_l\}\rangle \quad (104)$$

in place of $|\Psi(0)\rangle\rangle$ given by (78), with

$$|\{S_l;\theta_l,\phi_l\}\rangle = \prod_l|S_l;\theta_l,\phi_l\rangle, \quad (105)$$

we have

$$\langle I^-(t)\rangle \equiv \langle\Psi(0)|I^-(t)|\Psi(0)\rangle = ab^* e^{-i\omega_I t}\int dx\,|\psi(x)|^2\prod_l\left[\cos\frac{\Theta_l(x;t)}{2} - i\sin\theta_l\cos(\phi_l-\omega_l x/v)\sin\frac{\Theta_l(x;t)}{2}\right]^{2S_l}. \quad (106)$$

This is the desired result describing spin dephasing process of the incident particle in the fixed $\{S_l\}$ subspace of the detector spins.

In these expressions, the angle $\Theta_l(x;t)$ is an important quantity which describes the interaction between the incident particle and the detector.

From (34) this is given by

$$\Theta_l(x;t) = \int_0^t \mathrm{d}t'\, \Omega_l(x + vt' - x_l) = \frac{1}{v}\int_x^{x+vt} \mathrm{d}x'\, \Omega_l(x' - x_l). \tag{107}$$

In view of (80) or (106), it is evident that the quantity $x$ is associated with the orbital wave function $\psi(x)$ which is localized around the origin ($x = 0$) with a width $\Delta$. While in (107), $\Omega_l(x' - x_l)$ is assumed to be localized around $x' - x_l$ with a width $\delta$. Therefore, contribution to the integral in (107) comes mainly from the region of $x_l - \delta \le x' \le x_l + \delta$, allowing us to rewrite $\Theta_l(x;t)$ as

$$\Theta_l(x;t) = \frac{1}{v}\int_{-\infty}^{x+vt} \mathrm{d}x'\, \Omega_l(x' - x_l) \tag{108}$$

as far as the condition $d > c\delta$ is satisfied, $c$ being a suitable numerical factor.

Hence, we have

$$\Theta_l(x;t) = \Theta_l(x + vt) \tag{109}$$

where

$$\Theta_l(x) = \frac{1}{v}\int_{-\infty}^{x} \mathrm{d}x'\, \Omega_l(x' - x_l). \tag{110}$$

From the general expression (106) we can examine an ideal case where, at $t = 0$, the incident spin direction is parallel to $x$-axis, that is,

$$|I\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \tag{111}$$

and all the detector spins are in the down states, $|\{S_l;\theta_l = \pi, \phi_l = 0\}\rangle$.

Moreover, distance between adjacent spins in the detector is fixed to be $d$:

$$x_l = \Delta + ld \quad (l = 1, 2, ..., N) \tag{112}$$

where $\Delta$ is the width of $|\psi(x)|^2$ and we set

$$\Omega_l(x - x_l) = \frac{\Omega_l d}{\sqrt{2\pi\delta^2}} \mathrm{e}^{-(x-x_l)^2/2\delta^2}. \tag{113}$$

With an assumption that $\delta \gg \Delta$, we may put

$$|\psi(x)|^2 = \delta(x). \tag{114}$$

In this idealized case, we have from (107):

$$\langle I^x(t)\rangle = \frac{1}{2}\cos\omega_{\mathrm{I}}t \prod_l \left[\cos\frac{\Theta_l(vt)}{2}\right]^{2S_l}, \tag{115}$$

$$\langle I^y(t)\rangle = \frac{1}{2}\sin\omega_{\mathrm{I}}t \prod_l \left[\cos\frac{\Theta_l(vt)}{2}\right]^{2S_l}, \tag{116}$$

$$\langle I^z(t)\rangle = 0, \tag{117}$$

where $\Theta_l(x)$ is given by (110). This coincides with the ones for reference [32].

The last expression is obvious from

$$[I^z, \mathcal{H}] = 0 \tag{118}$$

where $\mathcal{H}$ is given by (78), and

$$\langle I^z(t)\rangle = \frac{1}{2}\left(|a|^2 - |b|^2\right). \tag{119}$$

### 3.2 Dynamics of the detector

Next we proceed to analyze dynamics of the detector. Relevant quantities are $\langle\langle S_l^-(t)\rangle\rangle$ and $\langle\langle S_l^z(t)\rangle\rangle$ defined by

$$\langle\langle S_l^-(t)\rangle\rangle = \langle\langle\Psi(0)|\mathrm{e}^{\mathrm{i}\mathcal{H}t/\hbar} S_l^- \mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}|\Psi(0)\rangle\rangle \tag{120}$$

and

$$\langle\langle S_l^z(t)\rangle\rangle = \langle\langle\Psi(0)|\mathrm{e}^{\mathrm{i}\mathcal{H}t/\hbar} S_l^z \mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}|\Psi(0)\rangle\rangle. \tag{121}$$

Thus it is convenient to evaluate the state vector

$$|\Psi(t)\rangle\rangle \equiv \mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}|\Psi(0)\rangle\rangle \tag{122}$$

where $|\Psi(0)\rangle\rangle$ is given by (78).

We first resolve the time evolution operator to obtain

$$\begin{aligned}|\Psi(t)\rangle\rangle &= \mathrm{e}^{-\mathrm{i}(\mathcal{H}_0+\mathcal{H}_1)t/\hbar}\mathcal{P}_+|\Psi(0)\rangle\rangle \\ &\quad + \mathrm{e}^{-\mathrm{i}\mathcal{H}_0 t/\hbar}\mathcal{P}_-|\Psi(0)\rangle\rangle \\ &= \mathcal{D}^z(\{\omega_l X/v\})\mathrm{e}^{-\mathrm{i}\mathcal{H}_0' t/\hbar}\left\{V(t)\mathcal{P}_+ + \mathcal{P}_-\right\} \\ &\quad \times \mathcal{D}^z(\{\omega_l X/v\})^\dagger|\Psi(0)\rangle\rangle\end{aligned} \tag{123}$$

where use has been made of equations (17, 35), $V(t)$ being given by (33).

Noting (92) we can successively apply the transformation property (67) on the spin coherent state $|\{\mathbf{z}_l\}\rangle$ of $|\Psi(0)\rangle\rangle$, (78), to find

$$\begin{aligned}|\Psi(t)\rangle\rangle &= \int \mathrm{d}x\, \psi(x)|x + vt\rangle \\ &\otimes\left\{a\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{I}}t/2}|+\rangle \otimes |\{\mathbf{z}_l^{(\mathrm{int})}(x)\}\rangle + b\mathrm{e}^{\mathrm{i}\omega_{\mathrm{I}}t/2}|-\rangle \otimes |\{\mathbf{z}_l^{(\mathrm{non})}\}\rangle\right\}\end{aligned} \tag{124}$$

where

$$\mathbf{z}_l^{(\mathrm{int})}(x) = \mathcal{R}_l^z\left(\omega_l\left(\frac{x}{v} + t\right)\right)\mathcal{R}_l^x\left(\Theta_l(x;t)\right)\mathcal{R}_l^z\left(-\omega_l x/v\right)\mathbf{z}_l$$
$$= \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\omega_l t/2}\left(z_{l,+}\cos\frac{\Theta_l(x;t)}{2} - \mathrm{i}z_{l,-}\mathrm{e}^{-\mathrm{i}\omega_l x/v}\sin\frac{\Theta_l(x;t)}{2}\right) \\ \mathrm{e}^{\mathrm{i}\omega_l t/2}\left(z_{l,-}\cos\frac{\Theta_l(x;t)}{2} - \mathrm{i}z_{l,+}\mathrm{e}^{\mathrm{i}\omega_l x/v}\sin\frac{\Theta_l(x;t)}{2}\right)\end{pmatrix} \tag{125}$$

and

$$\mathbf{z}_l^{(\mathrm{non})} = \mathcal{R}_l^z\left(\omega_l\left(\frac{x}{v} + t\right)\right)\mathcal{R}_l^z(-\omega_l x/v)\mathbf{z}_l = \begin{pmatrix} z_{l,+}\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{I}}t/2} \\ z_{l,-}\mathrm{e}^{\mathrm{i}\omega_{\mathrm{I}}t/2}\end{pmatrix}. \tag{126}$$

In obtaining these expressions, we used a relation of the form

$$\mathrm{e}^{-\mathrm{i}\xi P/\hbar}|x\rangle = |x+\xi\rangle. \tag{127}$$

Next, we note that

$$\langle \mathbf{z}_l^{(\mathrm{int})}|S_l^-|\mathbf{z}_l^{(\mathrm{int})}\rangle = \langle \mathbf{z}_l^{(\mathrm{int})}|b_{l,-}^\dagger b_{l,+}|\mathbf{z}_l^{(\mathrm{int})}\rangle = z_{l,-}^{(\mathrm{int})*} z_{l,+}^{(\mathrm{int})} \tag{128}$$

where use has been made of equations (36, 48).

Using equations (125, 73), we rewrite (128) as

$$\langle \mathbf{z}_l^{(\mathrm{int})}|S_l^-|\mathbf{z}_l^{(\mathrm{int})}\rangle = \frac{|z_l|^2}{2}\mathrm{e}^{-\mathrm{i}\omega_l t}\left[\left\{\mathrm{e}^{-\mathrm{i}\phi_l}\cos^2\frac{\Theta_l(x;t)}{2} + \mathrm{e}^{\mathrm{i}(\phi_l-2\omega_l x/v)}\sin^2\frac{\Theta_l(x;t)}{2}\right\}\sin\theta_l + i\mathrm{e}^{-\mathrm{i}\omega_l x/v}\sin\Theta_l(x;t)\cos\theta_l\right]. \tag{129}$$

Similarly we have

$$\langle \mathbf{z}_l^{(\mathrm{non})}|S_l^-|\mathbf{z}_l^{(\mathrm{non})}\rangle = \frac{1}{2}|z_l|^2\mathrm{e}^{-\mathrm{i}\omega_l t}\mathrm{e}^{-\mathrm{i}\phi_l}\sin\theta_l. \tag{130}$$

Consequently, with the use of (124), we can calculate (120) as

$$\langle\langle S_l^-(t)\rangle\rangle = \langle\langle \Psi(t)|S_l^-|\Psi(t)\rangle\rangle = \int \mathrm{d}x\,|\psi(x)|^2\left\{|a|^2\langle \mathbf{z}_l^{(\mathrm{int})}|S_l^-|\mathbf{z}_l^{(\mathrm{int})}\rangle + |b|^2\langle \mathbf{z}_l^{(\mathrm{non})}|S_l^-|\mathbf{z}_l^{(\mathrm{non})}\rangle\right\}. \tag{131}$$

We further rewrite (131), noting an identity

$$\frac{1}{2}|z_l|^2 = \mathrm{e}^{-|z_l|^2}\sum_{S_l=0,\frac{1}{2},\cdots}^{\infty}\frac{|z_l|^{4S_l}}{(2S_l)!}S_l, \tag{132}$$

to obtain

$$\langle\langle S_l^-(t)\rangle\rangle = \mathrm{e}^{-|z_l|^2}\sum_{S_l}\frac{|z_l|^{4S_l}}{(2S_l)!}S_l\mathrm{e}^{-\mathrm{i}\omega_l t}\int \mathrm{d}x\,|\psi(x)|^2 \times\left[\left\{|a|^2\left(\mathrm{e}^{-\mathrm{i}\phi_l}\cos^2\frac{\Theta_l(x;t)}{2} + \mathrm{e}^{\mathrm{i}(\phi_l-2\omega_l x/v)}\sin^2\frac{\Theta_l(x;t)}{2}\right) + |b|^2\mathrm{e}^{-\mathrm{i}\phi_l}\right\}\sin\theta_l + \mathrm{i}|a|^2\mathrm{e}^{-\mathrm{i}\omega_l x/v}\sin\Theta_l(x;t)\cos\theta_l\right]. \tag{133}$$

Referring to (77), we finally find the following result:

$$\langle S_l^-(t)\rangle \equiv \langle\Psi(0)|S_l^-(t)|\Psi(0)\rangle = \langle\Psi(t)|S_l^-|\Psi(t)\rangle \tag{134}$$

$$= S_l\mathrm{e}^{-\mathrm{i}\omega_l t}\int \mathrm{d}x\,|\psi(x)|^2 \times\left[\left\{|a|^2\left(\mathrm{e}^{-\mathrm{i}\phi_l}\cos^2\frac{\Theta_l(x;t)}{2} + \mathrm{e}^{\mathrm{i}(\phi_l-2\omega_l x/v)}\sin^2\frac{\Theta_l(x;t)}{2}\right) + |b|^2\mathrm{e}^{-\mathrm{i}\phi_l}\right\}\sin\theta_l + \mathrm{i}|a|^2\mathrm{e}^{-\mathrm{i}\omega_l x/v}\sin\Theta_l(x;t)\cos\theta_l\right] \tag{135}$$

where

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\mathcal{H}t/\hbar}|\Psi(0)\rangle, \tag{136}$$

$|\Psi(0)\rangle$ being given by (105), *i.e.*,

$$|\Psi(0)\rangle = |I\rangle\otimes|\psi\rangle\otimes|\{S_l;\theta_l,\phi_l\}\rangle. \tag{137}$$

The expression (135) gives details of the transverse dynamical process of the detector spin $\mathbf{S}_l$.

Similarly we can determine the longitudinal dynamical process by calculating $\langle\langle S_l^z(t)\rangle\rangle$:

$$\langle\langle S_l^z(t)\rangle\rangle = \mathrm{e}^{-|z_l|^2}\sum_{S_l}\frac{|z_l|^{4S_l}}{(2S_l)!}S_l\int \mathrm{d}x\,|\psi(x)|^2 \times\left[\left\{|a|^2\cos\Theta_l(x;t) + |b|^2\right\}\cos\theta_l + \mathrm{i}|a|^2\sin(\phi_l-\omega_l x/v)\sin\Theta_l(x;t)\sin\theta_l\right] \tag{138}$$

which gives immediately

$$\langle S_l^z(t)\rangle = S_l\int \mathrm{d}x\,|\psi(x)|^2\left[\left\{|a|^2\cos\Theta_l(x;t) + |b|^2\right\}\cos\theta_l + \mathrm{i}|a|^2\sin(\phi_l-\omega_l x/v)\sin\Theta_l(x;t)\sin\theta_l\right]. \tag{139}$$

As an ideal case of (114) with the same initial condition below (111), these are simplified to

$$\langle S_l^x(t)\rangle/S_l = -\frac{1}{2}\sin\omega_l t\,\sin\Theta_l(vt), \tag{140}$$

$$\langle S_l^y(t)\rangle/S_l = \frac{1}{2}\cos\omega_l t\,\sin\Theta_l(vt) \tag{141}$$

and

$$\langle S_l^z(t)\rangle/S_l = -\frac{1}{2}\left(\cos\Theta_l(vt)+1\right). \tag{142}$$

## 4 Numerical evaluations and concluding remarks

Let us examine the simplified case of (115–117). We show in Figure 1 time evolution of $\langle I^x(t)\rangle$ and $\langle I^y(t)\rangle$ for $N=5$, 10 and 20. For smaller values of $N$, numerical calculations were already done [32]. All the spin magnitude is assumed to be $S_l = 1/2$. The spin of the incident particle is largely perturbed when the particle passes through the detector region. For $N=5$, however, it is observed that the spin begins to oscillate freely after the particle left the detector region. With increase in $N$, both $\langle I^x(t)\rangle$ and $\langle I^y(t)\rangle$ tend to vanish when the particle passes through the detector region: this is clearly seen for $N=20$. Thus we have seen that the spin of the incident particle effectively loses its initial memory in the detector region for larger values of $N$. This is, of course, caused by the interaction between the two (the incident particle and the detector spins), but we should emphasize two points: one is that the phenomenon is observed when $N$ becomes large.

That is, the detector system plays effectively a role of reservoir with increasing values of $N$. The other is that the behavior in Figure 1 is also due to quantum fluctuations. Both the detector spins and an incident particle are quantum mechanical objects, and therefore their behavior is probabilistic in nature. This will clearly be seen when we find the so-called quasi-probability density. Details will be published in our following paper. Here, we only mention the fact that the quantum fluctuations are not only large but also multi-directional. Namely, probability to find the spins in other directions than the initially specified direction becomes rather large when the system evolves in time. Thus the whole system behaves in the probabilistic way. This is one of the reasons why we have the dephasing behavior of Figure 1. In Figure 2 we show time evolution of $\langle I^x(t)\rangle$ and $\langle I^y(t)\rangle$ for $S_1 = 1/2$, 5, 20 with $N = 1$. It is seen that dephasing characteristic can also be found for larger values of $S_1$. Thus, $S_l \rightarrow$ large gives qualitatively the similar effect as $N \rightarrow$ large even for a single detector spin, although the dephasing region is confined around $x_1$. This is due to increase in degree of freedom in the detector as $S_l$ and/or $N$ become large.

In Figure 3, we show time evolution of $\langle \mathbf{S}_1(t)\rangle$ by changing the interaction strength $\Omega_1$. When the incident particle comes around $x_1$, the detector spin $\mathbf{S}_1$ responds from the initial spin down state. As is seen from Figures 3a and 3b, after the passage of the particle, the spin $\mathbf{S}_1(t)$ oscillates freely. For the strong interaction case of Figure 3c, however, the free oscillation can no longer be seen. This is due to the strong perturbations caused by the incident particle destroying the initial coherence of $\mathbf{S}_1$ and overwhelming the effect of $\mathcal{H}_{\rm D}$. We have thus analyzed spin dynamics of the incident particle and the detector.

As mentioned in the introduction, our concern on the C–H model has been twofold; one is the decoherence process in a relevant system (incident particle for the C–H model) in strong interaction with its environment (detector) and the other is related with the quantum mechanical measurement process. As explicitly shown above, the decoherence process is seen to occur for larger values of $N$ and/or $S$. This can also be recognized as the collapse of wave function in the quantum mechanical measurement process which is the motif for the introduction of the C–H model. It should be stressed that the decoherence *process* is found in our work, namely, dynamical time evolution of the system is completely determined in contrast with the most of the previous theories whose main concern lies in the scattering matrix (*i.e.*, behavior at $t \rightarrow \infty$) [25–27,29, 30]. We are also interested in determining dynamical process of the detector to find $\langle \mathbf{S}_l\rangle_t$. As seen from Figure 3, the localized spin $\{\mathbf{S}_l\}$ indeed plays an essential role of detector. That is, the spin $\mathbf{S}_l$ makes a precise response when the incident particle passes nearby region around $x_l$. From the corresponding dynamics of $\mathbf{S}_l$, we can find information on the incident particle; we know when it enters and leaves the potential range of $\mathbf{S}_l$, and the interaction strength between the particle and the detector spin. These were done on the basis of the spin coherent state representation which enables us to treat the detector spins with arbitrary magnitude $\{S_l\}$. Even for $S_l \geq 1$, we can perform simple manipulations in the $2 \times 2$ space which is to be contrasted with the usual rather cumbersome ones in the angular momentum $(2S_l + 1)^2$ space [32].

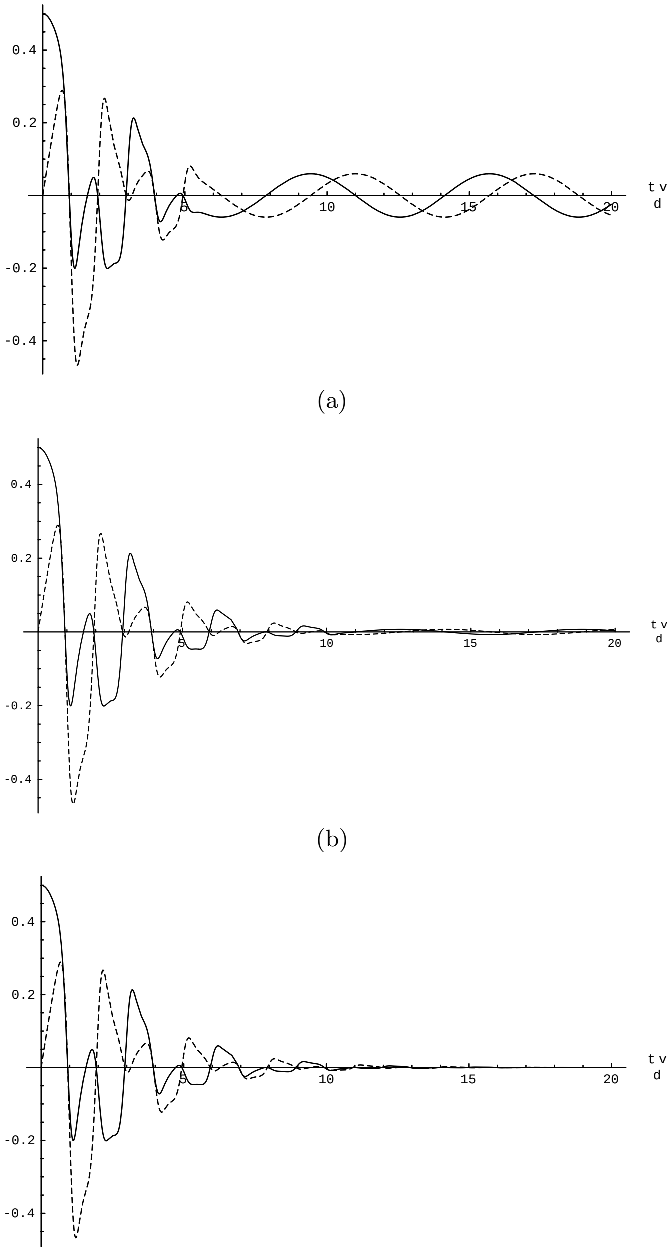


**Fig. 1.** Time evolution of $\langle \mathbf{I}(t)\rangle$ as a function of $tv/d$. The solid and broken lines correspond to $x$ and $y$-components, respectively, for the parameters $\hat{\Omega}_l \equiv \Omega_l d/v = 8$, $\delta/d = 0.25$, $S_l = 1/2$, and (a) $N = 5$, (b) $N = 10$, (c) $N = 20$.

Moreover, this formalism enables us rather easily to find a quasi-probability distribution treated in a forthcoming paper.

We should further emphasize that our "interim" expressions (97, 133, 138) incidentally solved a generalized version of the original Cini model [28]. Explicitly, the

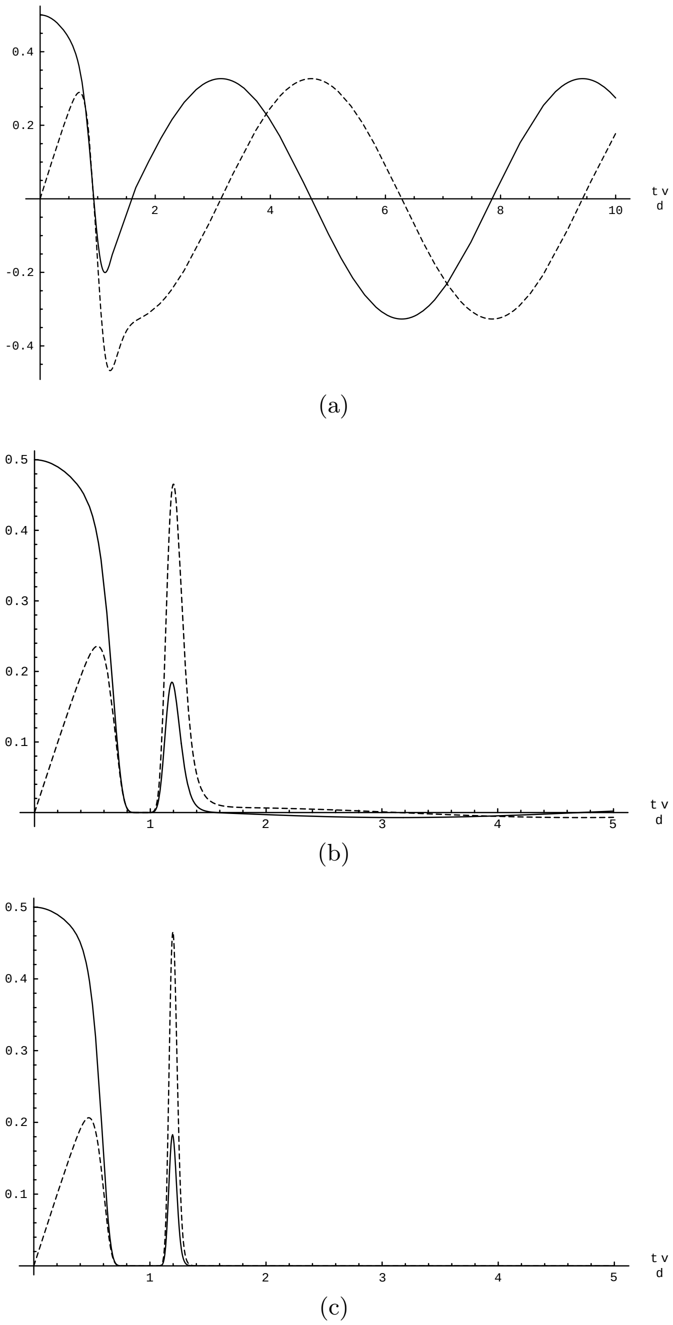


**Fig. 2.** Time evolution of $\langle \mathbf{I}(t) \rangle$ as a function of $tv/d$. The solid and broken lines correspond to $x$ and $y$-components, respectively, for the parameters $\Omega_l\, d/v = 8$, $\delta/d = 0.25$, $N = 1$, and (a) $S_1 = 1/2$, (b) 5, (c) 20.

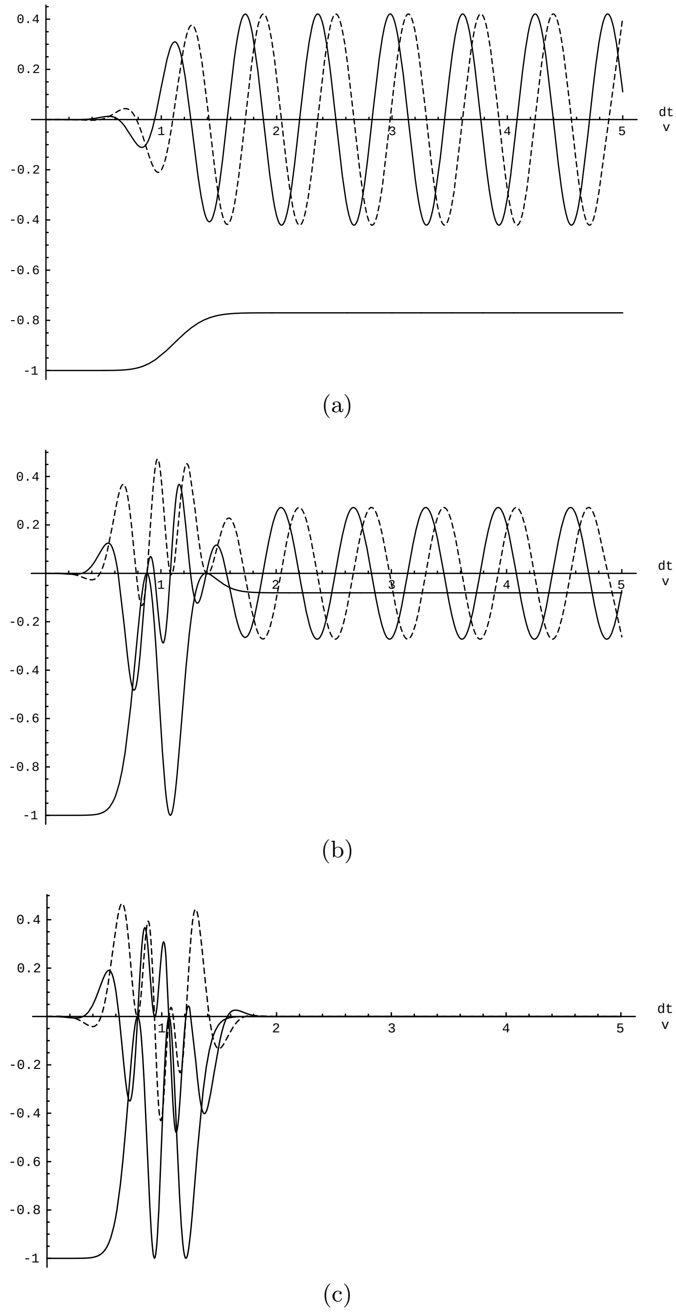


**Fig. 3.** Time evolution of $\langle \mathbf{S}_1(t) \rangle$ as a function of $tv/d$. The solid, and broken and thick lines correspond to $\langle S_1^x(t) \rangle / S_1$, $\langle S_1^y(t) \rangle / S_1$ and $\langle S_1^z(t) \rangle / S_1$, respectively, for $\delta/d = 0.25$, $\hat{\omega}_l \equiv \omega_l\, d/v = 10$, $S_1 = 1/2$. Case of (a) $\hat{\Omega}_l = 1$, (b) $\hat{\Omega}_l = 10$, (c) $\hat{\Omega}_l = 5\pi$.

generalized Cini model is obtained from (1) by the following replacement (see, Sect. 2.2.2) with $N = 1$:

$$S_l^{\pm} = b_{l,\pm}^{\dagger} b_{l,\mp}\ , \tag{143}$$

$$S_l^{z} = \left( N_{l,+} - N_{l,-} \right) / 2 \tag{144}$$

and

$$N_{l,\pm} = b_{l,\pm}^{\dagger} b_{l,\pm}, \tag{145}$$

where $b_{l,\pm}$ and $b_{l,\pm}^{\dagger}$ are boson operators. Details will be discussed in our subsequent papers where we also examine the dynamical processes of the Coleman-Hepp model by determining time evolution of a density matrix and quasi-probability density.

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